STUDENT WORKSHEET

# PENDULUMS and LAWS OF NATURE

## *Pendulums: Does nature always obey rules?*

*Investigating the Relationship Between Period and Length*

### INTRODUCTION

Many systems in nature can be described by pendulums. Using Newton's Laws of Motion and calculus, it can be shown that in a pendulum small displacements from the equilibrium position follow the laws of simple harmonic motion – where the restoring force on an object is proportional to its displacement from equilibrium. In the Searching series video clip you just watched, Dr. Alan Lightman describes the satisfaction of testing this law himself. Today, you will perform the same experiment.

### OBJECTIVE

In this lesson, you will collect and analyze data to determine the relationship between the length of a simple pendulum and its period of oscillation for small angles. After the lab, you'll reflect on your experimental process and explore the meaning of laws of nature.

### MATERIALS

- Lab stand with clamp
- String
- Bob or other small weight (washer, large bead, etc.)
- Meter stick or ruler
- Stopwatch
- Line of best fit ruler (optional)
- Excel (optional)

### PROCEDURE

1. **Set up the pendulum.**
   a. Attach the bob to the end of the shortest string.
   b. Hang the string from the clamp on the lab stand, ensuring the pendulum can swing freely.
   c. Measure the string from clamp to bob. Record the length in the table below.
2. **Prepare the first trial.**
   a. Using the protractor to measure, pull the pendulum back a small amount from the vertical. Keep the string taut.
3. **Complete the first trial.**
   a. Release the pendulum and start timing with the stopwatch. Let the pendulum complete 4 full swings before stopping the timer.
4. **Record the result.**
   a. Record the trial time and divide by 4 to calculate the period (T) of a single swing. Record the period in the table below.
5. **Complete two more trials with the same length of string.** Record your data in the table below.
6. **Repeat steps 1-5 with each string.**

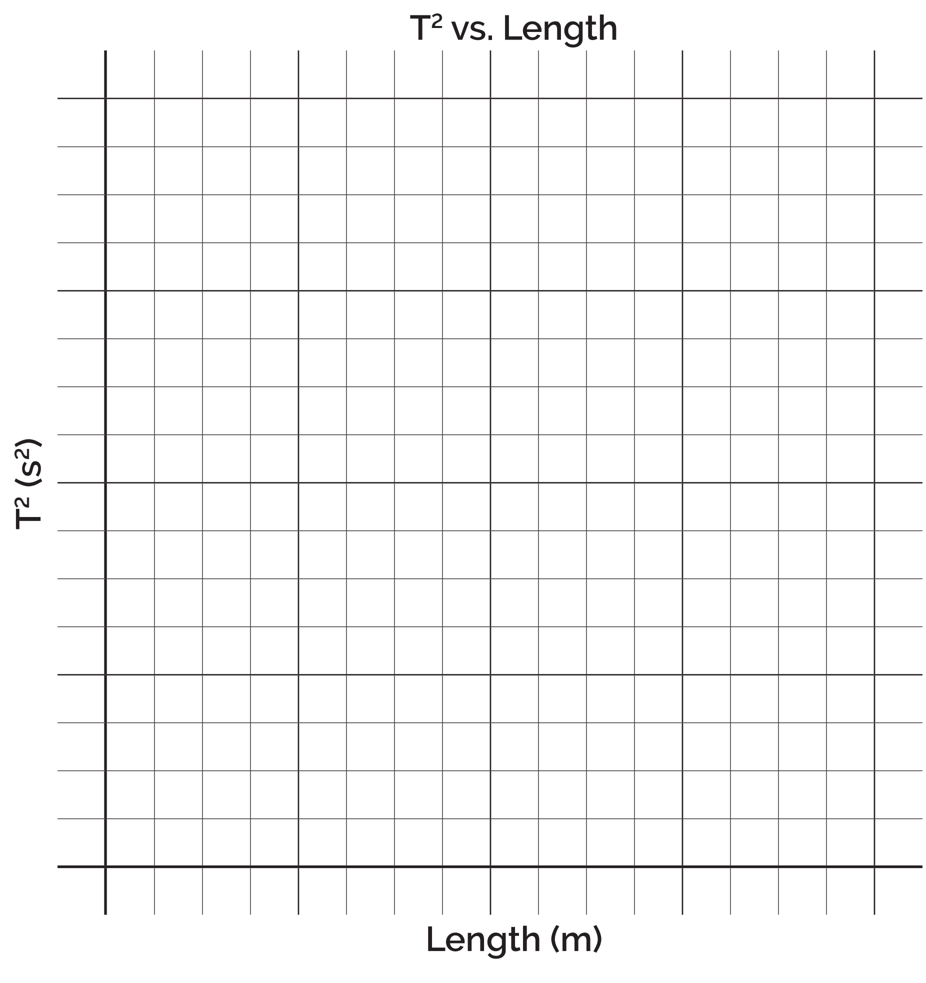
## DATA

| Length (m) | Trial 1 Period (s) | Trial 2 Period (s) | Trial 3 Period (s) | Average (s) | $T^2$ ($s^2$) |
|---|---|---|---|---|---|
| 0.10 | | | | | |
| 0.30 | | | | | |
| 0.50 | | | | | |

## ANALYZE DATA

*Graph*

1. Plot the square of the period vs. the length for each string on the graph below.
2. Using a line of best fit ruler, draw a trendline through the points on your graph below. Calculate the equation for the line. Trendline equation:____________________



*Analyze*

The mathematical relationship between the period and the length of a pendulum is given by:

$$T = 2\pi\sqrt{(L/g)}$$

1. **Determine the theoretical slope for the graph above. Show your work in the space below.**

2. **Calculate the percent error between the theoretical value for the slope of the line of best fit and your experimental value for the slope of the trendline you drew. Show your work in the space below.**

## CONCLUDE AND REFLECT

*Conclusion*

In the space below, write a paragraph conclusion which includes the following elements:

1. **Your experimental result**
2. **The mathematical relationship between pendulum length and period**
3. **Your percent error and possible sources of error**
4. **Your prediction about the relationship between other variables (i.e. mass, initial angle) and period**

*Reflection*

Using the questions below to guide your thoughts, reflect upon your experience during this lab. Jot down any thoughts in the space below and prepare to share your thoughts in a group discussion.

1. **What have you learned about the behavior of nature from this activity?**

2. **Do you think all physical phenomena in nature obey laws?**

3. **What do you think would happen if some natural phenomena did not obey laws?**

4. **How do laws of nature allow us to make predictions about physical phenomena?**

5. **Do you think that everything in nature follows laws of "cause and effect" – that is – every event that happens was caused by a previous event?**

## WORKSHEET FOR
# CALCULUS-BASED PHYSICS COURSES
(Optional)

## INTRODUCTION

A simple pendulum is a small mass suspended from a light, rigid rod. Ideally the mass will be small enough so that it will not deform the rod, but the rod should be light so that we can neglect its mass. In this lab, we will create a pendulum using a small mass bob and lightweight string. If we displace the bob slightly and let it go, the pendulum will start swinging. We will assume that there is no friction. In this idealized situation, once the pendulum is started, it will never stop. The only force acting on the pendulum is the force of gravity.

In the space below, draw a free-body diagram representing the forces present on the pendulum bob at the position shown in Figure A. Write an expression for the sum of forces in the x-direction, perpendicular to the string.

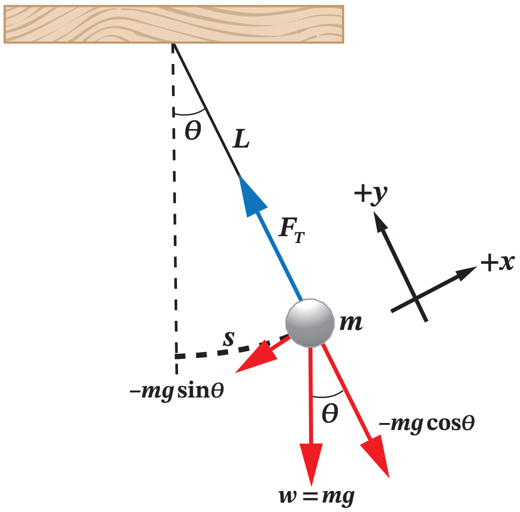



*Figure A*

$F_{net}^{x} =$

Consider Newton's 2nd law, $F_{net}^{x} = ma_x$ in the direction perpendicular to the string. Given the x-component of the force of gravity, our equation of motion for the pendulum is

$$-mg\sin\theta = ma$$
$$-g\sin\theta = a$$

Given $\frac{d^2\theta}{dt^2} = \frac{a}{L}$, rewrite the equation of motion in terms of $g$, $L$, and $\theta$.

For small angles (less than 15 degrees), $\sin\theta \approx \theta$. Using the small angle approximation gives an approximate solution of this differential equation for small angles,

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

and

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

This is a "harmonic oscillator" equation. In the space below, verify that the solution to $\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$ is $\theta = A\sin\left(\sqrt{(\frac{g}{L})}t\right)$ where $A$, the amplitude, is a constant and the largest angle that the pendulum achieves in its swing, determined by the initial conditions.

The time $t$ for the pendulum to make one complete swing, called the period $(\mathrm{T})$, is found by setting $\sqrt{(\frac{g}{L})}t = 2\pi$.

In the space below, solve for the period of one complete swing. This is the period equation of a simple pendulum.

$\mathrm{T} =$

Image source: *15.5: Pendulums - Physics LibreTexts*